HEAT TRANSFER OF AN ORTHOTROPIC BOUNDED CYLINDER UNDER BOUNDARY CONDITIONS OF THE FIRST AND THIRD KINDS

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Results are presented for investigating the two-dimensional nonstationary temperature field in an orthotropic bounded cylinder. A method is proposed for determining the ratio of the heat conductivity (thermal diffusivity) coefficients and the absolute values of the thermal diffusivity along the axes of a cylindrical coordinate system.

Investigations of the physicochemical properties of inhomogeneous and anisotropic structures are of practical importance. The thermal state of anisotropic media is described by a heat conduction equation of the form

$$c\gamma \frac{\partial T}{\partial \tau} = \lambda_{ij} T_{,ij}. \tag{1}$$

Analysis of different modifications of this equation is presented in [1]. For orthotropic media and independence of the thermophysical properties (TPP) from the temperature, (1) simplifies

$$\frac{\partial T}{\partial \tau} = a_i T_{,ii}, \tag{2}$$

i.e., the problem reduces to the solution of a two- or three-dimensional heat conduction equation. Such solutions can be found (for $a_i \equiv a$) in [1-4] and for distinct a_i in [5, 6].

Solutions for isotropic bodies during their heat transfer with a constant-temperature medium are examined in the monograph [2]. These solutions were the underpinnings of an experimental method to determine the thermal diffusivity coefficient of orthotropic bodies in a regular regime [7-9]. The equations (2) are complicated for the heat-transfer of bodies with media of different temperatures as well as in the presence of local thermal sources of two- and three-dimensional solutions, however the possibilities of practical applications of these solutions are expanded substantially [3, 4].

The analogy between (2) and the corresponding equation for an isotropic medium does not denote a one-to-one correspondence between the thermal states of orthotropic and isotropic bodies. Other conditions being equal, the temperature fields and thermal fluxes in orthotropic bodies depend substantially on the relationship between the TPP in different directions. However, there are no publications reflecting these singularities. In this connection the necessity occurred for investigation of the temperature field in an orthotropic body as a function of its properties.

It is shown in the example we examined that temperature field formation in orthotropic bodies has specifics inherent only in them, whose knowledge will permit a more rational organization of experimental TPP investigations for these materials and the necessity to correct the technology of their production. Let us present a formulation of the problem.

A bonded orthotropic cylinder is given with thermal diffusivity coefficients a_r and a_z in the directions of the r and z axes, respectively, of a cylindrical coordinate system whose origin is selected at the center of the cylinder. The initial temperature of the body mentioned is constant and equal to T_0. The cylinder altitude is 2h and the diameter is 2R. The cylinder surface end faces at the initial time are maintained at the temperature $T_c \neq$

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T_0, while heat transfer according to Newton's law occurs with a medium having the temperature T_0 at the side surface. It is necessary to find the two-dimensional nonstationary temperature field at any point of the orthotropic cylinder, i.e., to solve the equation

$$a_r\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right]+a_z\frac{\partial^2 T}{\partial z^2}=\frac{\partial T}{\partial \tau} \tag{3}$$

under the following initial and boundary conditions

$$T(r,\ z,\ 0)=T_0; \tag{4}$$

$$T(r,\ \pm h,\ \tau)=T_c; \tag{5}$$

$$\frac{\partial T(r,\ 0,\ \tau)}{\partial z}=0; \tag{6}$$

$$\frac{\partial T(R,\ z,\ \tau)}{\partial \tau}=-\frac{\alpha}{\lambda_r}[T(R,\ z,\ \tau)-T_0]. \tag{7}$$

A solution of (3) under the conditions (4)-(7), obtained by using the Laplace and Hankel integral transforms has the form

$$\begin{aligned}\Theta^*(r,\ z,\ \tau)=1-2\sum_{m=1}^{\infty}A_m J_0\left(\delta_m\frac{r}{R}\right)\frac{\operatorname{ch}\left(\delta_m K\sqrt{K_a}\,\frac{z}{h}\right)}{\operatorname{ch}(\delta_m K\sqrt{K_a})}+\\+4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(-1)^{n+1}A_m J_0\left(\delta_m\frac{r}{R}\right)\frac{\mu_n\cos\left(\mu_n\frac{z}{h}\right)}{\mu_n^2+\delta_m^2K_aK^2}\times\\\times\exp[-(\mu_n^2+\delta_m^2K_aK^2)\,\mathrm{Fo}_h],\end{aligned} \tag{8}$$

where

$$A_m=J_1(\delta_m)/\delta_m[J_0^2(\delta_m)+J_1^2(\delta_m)];$$

δ_m, μ_n are roots, respectively, of the characteristic equations

$$\frac{J_0(\delta_m)}{J_1(\delta_m)}=\frac{\delta_m}{\mathrm{Bi}_R}; \tag{9}$$

$$\cos\mu_n=0. \tag{10}$$

The expression (8) obtained consists of two parts: the stationary component Θ^*_{st} and the nonstationary component, i.e., $\Theta^*(r,\ z,\ \tau)=\Theta^*_{st}(r,\ z)+\Theta^*_{nonst}(r,\ z,\ \tau)$, where

$$\begin{aligned}\Theta^*_{st}(r,\ z)=1-2\sum_{m=1}^{\infty}A_m J_0\left(\delta_m\frac{r}{R}\right)\frac{\operatorname{ch}\left(\delta_m K\sqrt{K_a}\,\frac{z}{h}\right)}{\operatorname{ch}(\delta_m K\sqrt{K_a})}=\\=f(\mathrm{Bi}_R,\ K,\ K_a);\end{aligned} \tag{11}$$

$$\begin{aligned}\Theta^*_{nonst}(r,\ z,\ \tau)=4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(-1)^{n+1}A_mJ_0\left(\delta_m\frac{r}{R}\right)\frac{\mu_n\cos\left(\mu_n\frac{z}{h}\right)}{\mu_n^2+\delta_m^2K_aK^2}\times\\\times\exp[-(\mu_n^2+\delta_m^2K_aK^2)\,\mathrm{Fo}_h]=f(\mathrm{Bi}_R,\ K,\ K_a,\ \mathrm{Fo}_h).\end{aligned} \tag{12}$$

The expression (8) can be used to solve the most diverse applied problems: to compute the temperature fields, heat fluxes, and also to determine the thermophysical properties of real objects in the shape of bounded orthotropic cylinders.

The whole heat transfer process can evidently (see (8)) be separated into three stages: initial (purely nonstationary), regular, and stationary.

In a particular case (under the same boundary conditions and $K_a = 1$), a solution for a bounded cylinder [3, 4] and a solution for an unbounded plate ($R \to \infty$), which is analyzed in detail in [2], results from (8).

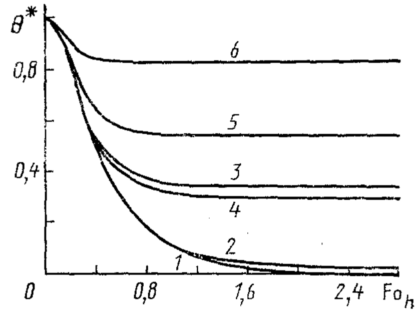

Fig. 1. The dependence $\Theta^*(0, 0, \tau) = f(Fo_h)$ for $Bi_R = 10$: 1) $K_a = 0.1$, $K = 0.2$; 2) 2 and 0.2; 3) 10 and 0.2; 4) 1 and 0.6; 5) 2 and 0.6; 6) 5 snf 0.6

TABLE 1. Values of the Stationary Temperature Θ^*_{st} at Points of the z = 0 Plane for Different Values of the Parameter K_0 and the Biot Criterion

Bi_R	Ko	Θ^*_{st}					
		$r/R=0$	$r/R=0,2$	$r/R=0,4$	$r/R=0,6$	$r/R=0,8$	$r/R=1$
∞	0,01	0,0000	0,0000	0,0000	0,0000	0,0009	1,0000
	0,05	0,0000	0,0000	0,0000	0,0000	0,0026	1,0000
	0,1	0,0000	0,0000	0,0002	0,0031	0,0616	1,0000
	0,2	0,0034	0,0059	0,0187	0,0719	0,2932	1,0000
	0,3	0,0378	0,0489	0,0921	0,2062	0,4845	1,0000
	0,5	0,2322	0,2554	0,3313	0,4774	0,7104	1,0000
	1,0	0,7213	0,7354	0,7767	0,8411	0,9201	1,0000
	2,0	0,9739	0,9754	0,9795	0,9858	0,9930	1,0000
	5,0	0,9999	0,9999	0,9999	0,9999	1,0000	1,0000
	10	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
10	0,01	0,0000	0,0000	0,0000	0,0000	0,0002	0,0701
	0,05	0,0000	0,0000	0,0000	0,0000	0,0006	0,2806
	0,1	0,0000	0,0000	0,0001	0,0012	0,0244	0,4497
	0,2	0,0020	0,0034	0,0108	0,0415	0,1702	0,6407
	0,3	0,0257	0,0333	0,0627	0,1405	0,3347	0,7443
	0,5	0,1847	0,2032	0,2640	0,3827	0,5802	0,8532
	1,0	0,6619	0,6756	0,7161	0,7811	0,8644	0,9552
	2,0	0,9600	0,9618	0,9672	0,9753	0,9850	0,9951
	5,0	0,9999	0,9999	0,9999	1,0000	0,0000	1,0000
	10	1,0000	1,0000	1,0000	1,0000	0,0000	1,0000
0,1	0,01	0,0000	0,0000	0,0000	0,0000	0,0000	0,0009
	0,05	0,0000	0,0000	0,0000	0,0000	0,0000	0,0038
	0,1	0,0000	0.0000	0,0000	0,0001	0,0004	0,0076
	0,2	0,0000	0,0001	0 0002	0,0010	0,0040	0,0157
	0,3	0,0008	0,0010	0,0019	0,0043	0,0104	0,0244
	0,5	0,0087	0,0096	0,0124	0,0182	0,0281	0,0440
	1,0	0,0692	0,0708	0,0842	0,0842	0,0963	0,1122
	2,0	0,0766	0,2780	0,2822	0,2892	0,2989	0,3114
	5,0	0,7775	0,7780	0,7793	0,7814	0,7844	0,7882
	10	0,9753	0,9753	0,9754	0,9757	0,9760	0,9764

Realization of heat transfer in a constant-temperature medium results in total equilibration of the temperature over the body volume ($\Theta^*_{st} = 0$). In the case under consideration the heat transfer occurs with media of different temperatures (see (4)-(7)) and the stationary component may differ from zero.

It follows from an analysis of (8) that heat propagation in a radial direction (the ap-

TABLE 2. Values of the Stationary Temperature Θ^*_{st} at Points of Planes for Different z/h, Ko and Bi_R = 10

Ko	z/h	Θ^*_{st}					
		r/R=0	r/R=0,2	r/R=0,4	r/R=0,6	r/R=0,8	r/R=1
0,2	0	0,0020	0,0034	0,0108	0,0415	0,1702	0,6407
	0,2	0,0019	0,0032	0,0102	0,0395	0,1624	0,6302
	0,4	0,0015	0,0027	0,0087	0,0336	0,1391	0,5961
	0,6	0,0012	0,0020	0,0064	0,0244	0,1020	0,5278
	0,8	0,0006	0,0010	0,0033	0,0127	0,0540	0,3939
	1,0	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
2,0	0	0,9600	0,9618	0,9672	0,9753	0,9850	0,9951
	0,2	0,9440	0,9466	0,9539	0,9652	0,9789	0,9931
	0,4	0,8841	0,8891	0,9038	0,9266	0,9550	0,9852
	0,6	0,7404	0,7498	0,7784	0,8260	0,8902	0,9634
	0,8	0,4489	0,4602	0,4971	0,5701	0,6984	0,8922
	1,0	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

pearance of two-dimensionality) at any point of an orthotropic cylinder depends not only on the heat transfer conditions on the side surface and the ratio between the geometric dimensions but also on the relationship between the thermophysical properties, i.e., on the parameter $K_a = L_\lambda$. Such a dependence is looked over quite well in the graphs presented in Fig. 1. Where all the dependences for the point r = z = 0 for $K_a \leq 1$ and K = 0.2 are superposed on curve 1 that corresponds to the one-dimensional case and illustrates total equilibration of the temperature $(T(0, 0, \tau) \to T_c)$. For $K_a > 1$ the one-dimensionality at the same point is spoiled (curves 2 and 3). As K = h/R increases, the one-dimensionality is also spoiled (curve 4). However, even for large K but $K_a \ll 1$ the dependences $\Theta^*(0, 0, \tau)$ are superposed on the curve 1.

Therefore, for a given Bi_R the variation in the parameters K and K_a influences the temperature at any point of the body. Therefore, it is necessary to examine the complex $K\sqrt{K_a}$, entering essentially into the solution (8) and playing the same part as the parameter K in the two-dimensional solution for an isotropic bounded cylinder.

Let us call the complex $Ko = K\sqrt{K_a}$ the two-dimensionality criterion for the temperature of an orthotropic cylinder of finite size. Then, it can be asserted on the basis of [3] that for any Bi_R for Ko < 0.25 on the r = 0 axis in an orthotropic bounded cylinder the excess temperature $\Theta^*(0, z, \tau)$, including the stationary state also, will be described with a sufficiently small error (0.4%) by the one-dimensional solution for an unbounded plate [2]. This later situation is confirmed by data presented in Tables 1 and 2. By using these tables a comparative estimate can be made about the agreement of the two-dimensional stationary solution (11) for certain points of a finite orthotropic cylinder for different values of the criteria Ko, Bi_R and the one-dimensional solution for an unbounded plate $(\Theta^*_{nonst}$ for different r.

Let us examine the solution (8) in the limit cases when $K_a \to 0$ and $K_a \to \infty$:

$$\lim_{K_a \to 0} \Theta^*(r, z, \tau) = 2 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos\left(\mu_n \frac{z}{h}\right)}{\mu_n} \exp(-\mu_n^2 Fo_h), \tag{13}$$

$$\lim_{K_a \to \infty} \Theta^*(r, z, \tau) = \begin{cases} 0 & \text{for } z = h; \\ 1 & \text{for } 0 \leqslant z < h. \end{cases} \tag{14}$$

We therefore obtain that as $K_a \to 0$ $(a_r \ll a_z)$ the temperature field over the whole cylinder volume is described by the one-dimensional solutions, i.e., in this case the orthotropic solution cylinder is an ideal heat conductor for any heat transfer conditions on the side surface, which will be absent in this case.

As $K_a \to \infty$ $(a_r \gg a_z)$ the inner layers of the orthotropic cylinder do not heat up and all the heat "is leaked" to the endface surface. In this case a bounded orthotropic cylinder is an ideal heat insulator (refractory).

Analyzing (8) and (14), the deduction can be made that as K_a $(K_a > 1)$ grows, the height of the cylinder inner layer adjoining the z = h plane where the heating occurs (the temperature is different from the initial value) diminishes. For sufficiently large K_a the height of this layer tends to zero.

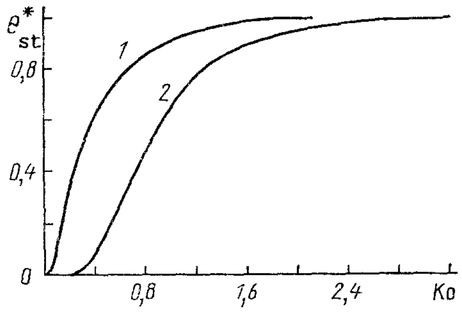

Fig. 2

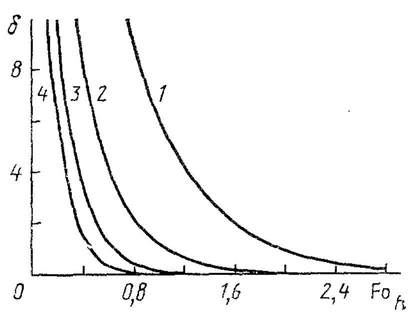

Fig. 3

Fig. 2. The dependence $\Theta^*_{st}(r, 0) = f(Ko)$ for $Bi_R = 10$; 1) $r/R = 0.9$; 2) 0.

Fig. 3. Dependence of the relative contribution of the first term of the series (12) to the sum of the whole series in percents for $Ri_R = 10$ ($\delta\% = f(Fo_h)$: 1) Ko = 0.3, 2) 0.4, 3) 0.5, 4) 0.6.

On the other hand, for sufficiently small K_a ($K_a < 1$) equilibration of the temperature to T_c at all points of the planes z/h occurs identically (see (13)). This means that only in a certain band can K_a be determined (for given K) from the results of temperature measurements at given points of an orthotropic cylinder. The boundaries of this K_a band depend substantially on the values of the parameter K and the coordinates of the temperature measurement points.

Analyzing the dependences in Fig. 2, it can be seen that for an 0.5% accuracy in measuring $\Theta^*_{st}(r, z)$, the band of Ko determination will lie within the limits 0.25-2.8 for the point $r/R = 0$ and within the limits 0.04-2 for $r/R = 0.9$. The passage from bands in Ko to bands in K_a as a function of the value of K is made by means of the formula

$$K_a = \frac{Ko^2}{K^2}. \tag{15}$$

Numerical data on the bands to determine K_a as a function of the parameter K for different r/R are presented in Table 3. It is seen from the table that if we select K = 0.2 then taking the temperature measurement at two points into account ($r/R = 0$ and $r/R = 0.9$), the band to determine K_a will be 0.04-196. For given K and known Bi_R evidently $Bi_R\Theta^*_{st}$ will be a function of just K_a at any point.

Therefore, in a stationary thermal regime there is the possibility of determining the criterion Ko since $\Theta^*_{st}(r, z) = F(Bi_R, Ko)$ from (11). However, it is first necessary to know the quantity Bi_R that can easily be found experimentally in the regular regime [10, 11].

Let us consider the nonstationary component (12) of the solution (8). The double series (12) is rapidly convergent and for definite values of the criteria Ko and Fo_h can be limited to the first term of this series with a sufficient degree of accuracy, i.e., the regular heat transfer regime sets in. The time of regular regime onset (τ_p) depends substantially on the value fo Ko: the greater the magnitude of this criterion, the smaller the time τ_p.This is seen well from the graphs presented in Fig. 3. The graphs are dependences of the relative contribution of the first term of the series (12) to the sum of the whole series in percents (for $Bi_R = 10$), i.e.,

$$\delta\,\% = \left| \frac{\Theta^*_{nonst}(\Sigma) - \Theta^*_{nonst}(1\,\text{term})}{\Theta_{nonst}(\Sigma)} \right| \cdot 100\% = f(Bi_R,\ Ko, Fo_h). \tag{16}$$

Data are presented in [3] about the time of regular thermal regime onset for isotropic bodies of the simples (classical) shape for heat transfer in a constant-temperature medium as a function of the ratio of the geometric dimensions K*. Other conditions being equal, the regularization time in an anisotropic medium depends substantially on the ratio of the thermophysical properties. In particular, for an orthotropic bounded cylinder, this dependence is seen from the determination of the criterion $Ko = K\sqrt{K_a}$, if K has a fixed value.

Let us note that practically all the data on the heat transfer investigation in a

TABLE 3. Ranges of K_a Determination for Different r/R as a Function of the Parameter K (Bi_R = 10)

K	$r/R=0$		$r/R=0,9$		K	$r/R=0$		$r/R=0,9$	
	Ko=0,25	Ko=2,8	Ko=0,04	Ko=2		Ko=0,25	Ko=2,8	Ko=0,04	Ko=2
	$K_{a\min}$	$K_{a\max}$	$K_{a\min}$	$K_{a\max}$		$K_{a\min}$	$K_{a\max}$	$K_{a\min}$	$K_{a\max}$
0,05	25	3136	0,64	1600	1,00	0,06	7,84	0,0016	4
0,10	6,25	784	0,16	400	1,10	0,05	6,48	0,0013	3
0,20	1,56	196	0,04	100	1,20	0,04	5,44	0,0011	2,8
0,30	0,69	87	0,018	44	1,30	0,037	4,64	0,0009	2,4
0,40	0,39	49	0,01	25	1,50	0,03	3,48	0,0007	1,8
0,50	0,25	31	0,006	16	2,00	0,016	1,96	0,0004	1
0,60	0,17	21,7	0,004	11	2,50	0,01	1,25	0,00026	0,64
0,70	0,13	16	0,003	8	3,00	0,007	0,87	0,00018	0,44
0,80	0,10	12,25	0,0025	6	5,00	0,0025	0,3	0,00006	0,16
0,90	0,08	9,7	0,002	5					

bounded isotropic cylinder can be utilized when studying the heat transfer of an orthotropic cylinder. In this case, values of the quantity Ko must be understood by the quantities K = h/R.

Returning to the investigation of the regular thermal regime, let us note that the dependence of the expression (16) on the criterion Bi_R is not essential and, consequently, is not considered within the framework of this paper.

Therefore, in a regular thermal regime the series (12) can be replaced by its first term, i.e.,

$$\Theta^*_{\text{nonst}}(r,\ z,\ \tau) = \Theta^*(r,\ z,\ \tau) - \Theta^*_{\text{st}}(r,\ z) =$$

$$= 4\,\frac{J_0\left(\delta_1 \dfrac{r}{R}\right) J_1(\delta_1)}{\delta_1\,[J_0^2(\delta_1) + J_1^2(\delta_1)]}\;\frac{\mu_1 \cos\left(\mu_1 \dfrac{z}{h}\right)}{(\mu_1^2 + \delta_1^2\,\mathrm{Ko}^2)}\exp[-(\mu_1^2 + \delta_1^2\,\mathrm{Ko}^2)\,\mathrm{Fo}_h] = f(\delta_1,\ \mathrm{Ko}). \tag{17}$$

Then the ratio of Θ^*_{nonst} at two different points of an orthotropic cylinder (for instance, $r_1 = z = 0$, $r_2 = 0.9R$, $z = 0$) at the identical time $\tau_1 > \tau_p$ will be a function of just the first root of the characteristic equation (9):

$$\Delta\Theta = \frac{\Theta^*_{\text{nonst}}(0,\ 0,\ \tau_1)}{\Theta^*_{\text{nonst}}(r_2,\ 0,\ \tau_1)} = \frac{1}{J_0(0{,}9\cdot\delta_1)} = f(\delta_1). \tag{18}$$

Now δ_1 can be found from (18) and then by using (9) the criterion Bi_R can be determined, and therefore, the roots of (9) also. If $Bi_R = \infty$, then the roots δ_m are found from $J_0(\delta_m) = 0$.

The heating (cooling) tempo will be determined by the expression

$$m^* = \frac{\ln\left[\dfrac{\Theta^*_{\text{nonst}}(0,\ 0,\ \tau_1)}{\Theta^*_{\text{nonst}}(0,\ 0,\ \tau_2)}\right]}{\tau_2 - \tau_1} = \left(\frac{\pi^2}{4} + \delta_1^2\,\mathrm{Ko}\right)\frac{a_z}{h^2}\ , \tag{19}$$

where $\tau_2 > \tau_1 > \tau_p$.

Starting from the above exposition, two modifications are proposed for the determination of the ratio $K_a = K_\lambda$ and the thermal diffusivity coefficients a_z and a_r from the results of measuring the excess temperature at two points (r = 0, r = 0.9) of the z = 0 plane of an orthotropic cylinder.

1. If the excess temperature is zero at the point r = z = 0 in the stationary regime (i.e. is described by a one-dimensional solution) and is different from zero (a two-dimensional solution) at the point r = 0.9R, z = 0, then we find a_z from (13). We determine the first root δ_1 of the characteristic equation (9) for the point r = 0.9R, z = 0 in the regular thermal regime by using (18). Then we determine the criterion Ko from (19) and using (15) we find K_a. The thermal diffusivity coefficient along the r axis is determined from the formula $a_r = K_a a_z$.

2. If the excess temperature at the point r = z = 0 in the stationary regime is not zero (i.e., is described by a two-dimensional solution), then it is first necessary to determine δ_1 in the regular thermal regime at the two points r = 0, r = 0.9R; z = 0 by using (18) and then the m roots of (9). Afterwards we find Ko and K_a by using (11) and a_z from (19), and we determine a_r as in the preceding case.

Let us examine the modification when the excess temperatures in the two points mentioned equal each other and equal zero. This means that $a_z \gg a_r$ and only determination of a_z is possible. If these excess temperatures equal each other and equal the value $T_c - T_0$, this means that $a_r \gg a_z$ (see (14)) and determination of the thermophysical properties is impossible in this case. The above is characteristic for any plane $0 \le z < h$.

The magnitude of the ratio K_a can also be assessed from the value of the stationary excess temperature at the point $r = z = 0$ by selecting K in such a manner that the condition of one-dimensionality of the heat propagation would still be satisfied at this point for $K_a = 1$. Then if the excess temperature equals zero, $K_a = 1$. If the excess temperature is not zero then $K_a > 1$.

In conclusion, let us note that one specimen is required to determine the ratio of the thermophysical properties and the thermal diffusivity coefficients a_r and a_z in contrast to the method presented in [7-9].

NOTATION

$\Theta(r, z, \tau) = T(r, z, \tau) - T_0$, excess temperature, $\vartheta^*(r, z, \tau) = [T_c - T(r, z, \tau)]/J(T_c - T_0)$, dimensionless excess temperature; τ, τ_p, running time and time of regular thermal regime onset; r, z, running coordinates; $k = h/R$, ratio between the geometric dimensions of the orthotropic cylinder; λ_r, λ_z, a_r, a_z, heat conductivity and thermal diffusivity coefficients, respectively, in the r and z axis directions; $K_z = K_\lambda = \lambda_r/\lambda_z = a_r/a_z$, ratio of the thermophysical properties; $Ko = K\sqrt{K_a}$, criterion for two-dimensionality of the orthotropic cylinder temperature; $Bi_R = \alpha R/\lambda_r$, Biot criterion; $Fo_h = a_z\tau/h^2$, Fourier criterion, and $J_0(x)$, $J_1(x)$, Bessel functions of the first kind.

LITERATURE CITED

1. H. Carslaw and D. Jaeger, Conduction of Heat in Solids, Oxford Univ. Press, NJ (1959).
2. A. V. Lykov, Theory of Heat Conduction [in Russian], Moscow (1967).
3. A. G. Shashkov, G. M. Volokhov, T. N. Abramenko, and V. P. Kozlov, Methods of Determining the Thermal Diffusivity and Thermal Conductivity [in Russian], Moscow (1972).
4. B. P. Kozlov, Two-dimensional Axisymmetric Nonstationary Heat Conduction Problems [in Russian], Minsk (1986).
5. I. A. Motovilovets and V. I. Kozlov, Mechanics of Coupled Fields in Structure Elements. Vol. 1. Thermoelasticity [in Russian], Kiev (1987).
6. A. I. Zhornik and E. M. Kartashov, Izv. Vyssh. Uchebn. Zaved., Aviats. Tekh., No. 3, 128-132 (1973).
7. M. P. Emchenko, Investigations in the Area of Thermal Measurements [in Russian], Moscow–Leningrad (1956), pp. 40-48.
8. M. P. Emchenko, Izv. Vyssh. Uchebn. Zaved., Priborostr., No. 3, 117-124 (1958).
9. M. P. Emchenko, Tr., LTA, No. 83, 203-222 (1959).
10. G. M. Kondrat'ev, Regular Thermal Regime [in Russian], Moscow–Leningrad (1954).
11. G. M. Kondrat'ev, Thermal Measurements [in Russian], Moscow–Leningrad (1957).